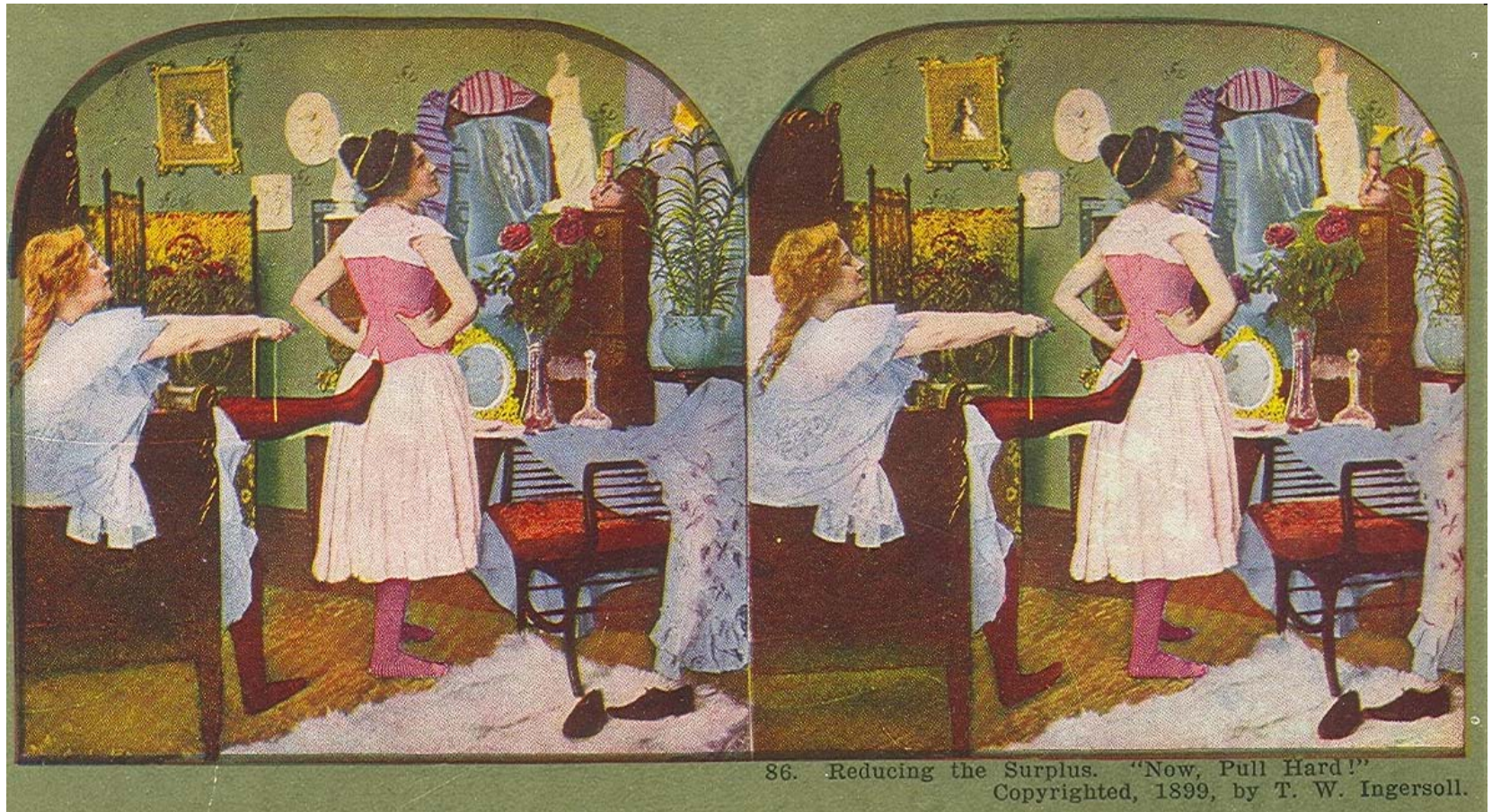
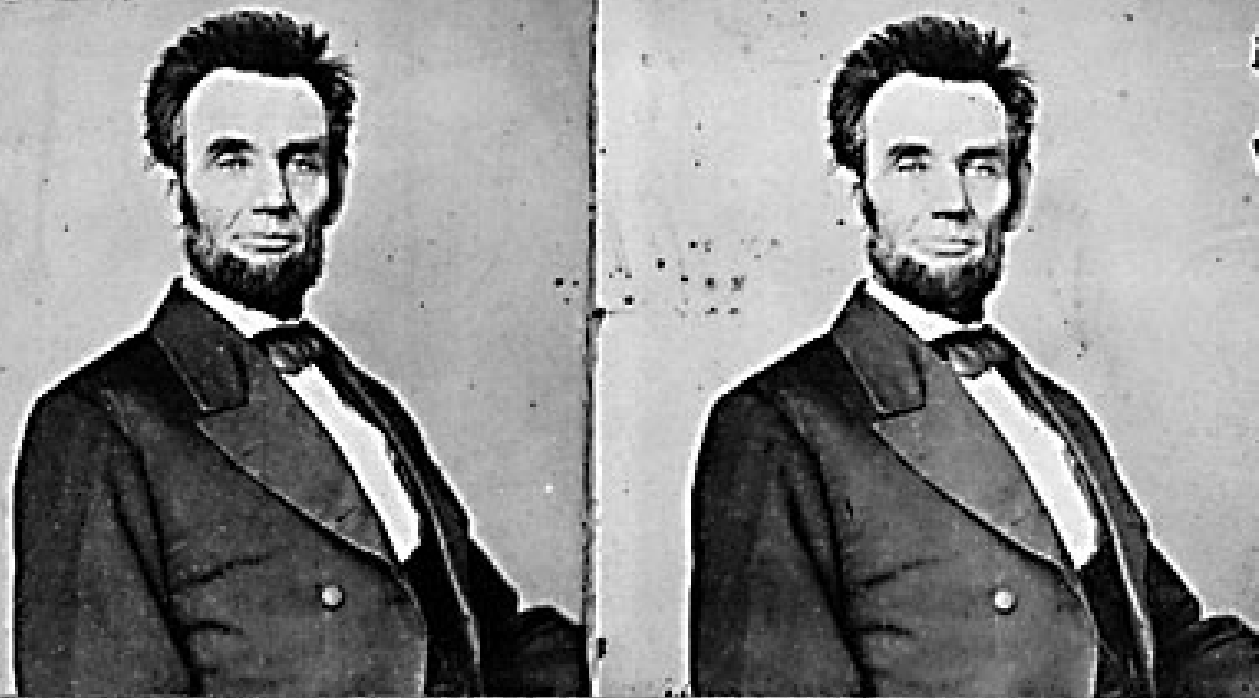


# Lecture 16

## Stereo and 3D Vision



HON. ABRAHAM LINCOLN, President of United States.



# Stereoscopes: A 19<sup>th</sup> Century Pastime

---





Public Library, Stereoscopic Looking Room, Chicago, by Phillips, 1923





Teesta suspension bridge-Darjeeling, India





Woman getting eye exam during immigration procedure at Ellis Island, c. 1905 - 1920 , UCR Museum of Photography





Mark Twain at Pool Table", no date, UCR Museum of Photography



# Anaglyphs

---

**Anaglyphs** provide a stereoscopic 3D effect when viewed with 2-color glasses (each lens a chromatically opposite color, usually red and cyan).

[http://en.wikipedia.org/wiki/Anaglyph\\_image](http://en.wikipedia.org/wiki/Anaglyph_image)

A free pair of red-blue stereo glasses can be ordered from [Rainbow Symphony Inc](#)

- <http://www.rainbowsymphony.com/freestuff.html>



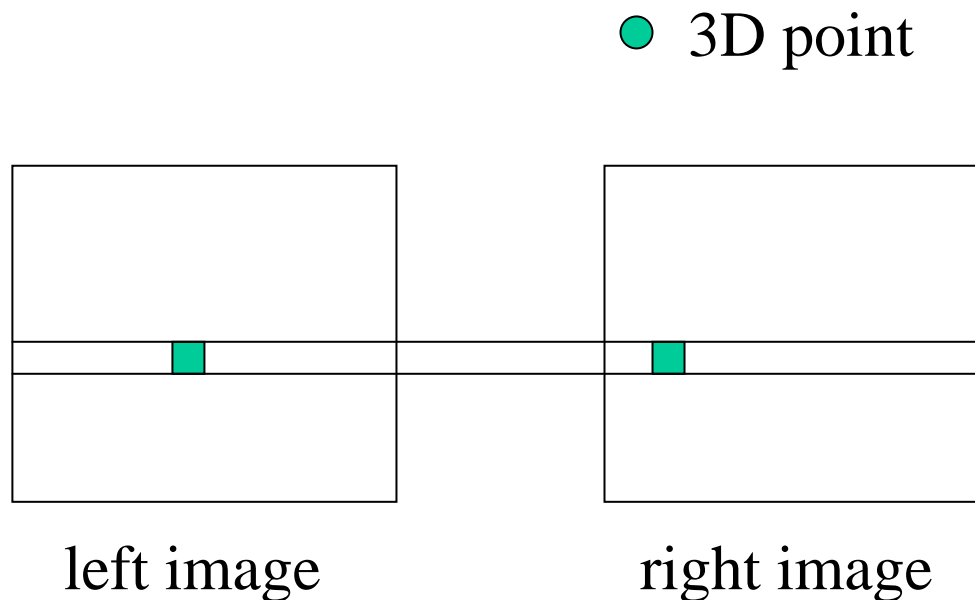
# 3D Movies



# How do we get 3D from Stereo Images?

---

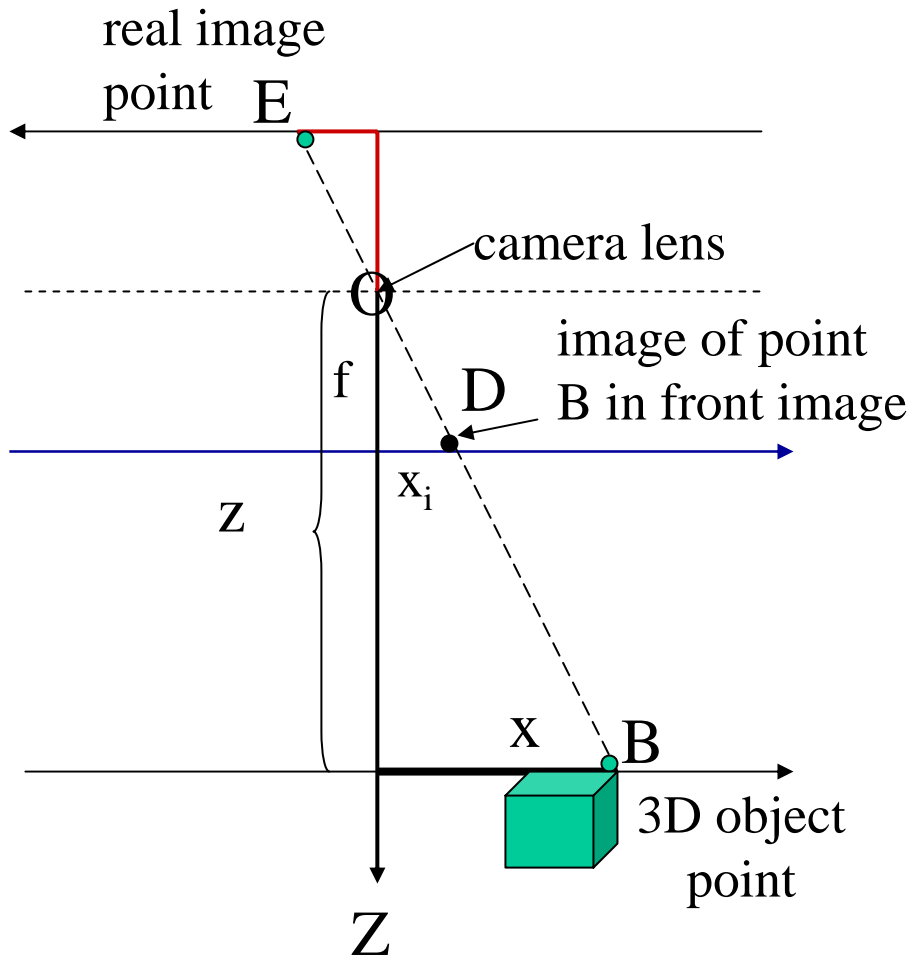
Perception of depth arises from “disparity” of a given 3D point in your right and left retinal images



disparity: the difference in image location of the *same 3D point* when projected under perspective to two different cameras

$$d = x_{\text{left}} - x_{\text{right}}$$

# Recall (from Lecture 5): Perspective Projection



This is the axis of the real image plane.

$O$  is the center of projection.

This is the axis of the **front image plane**, which we use.

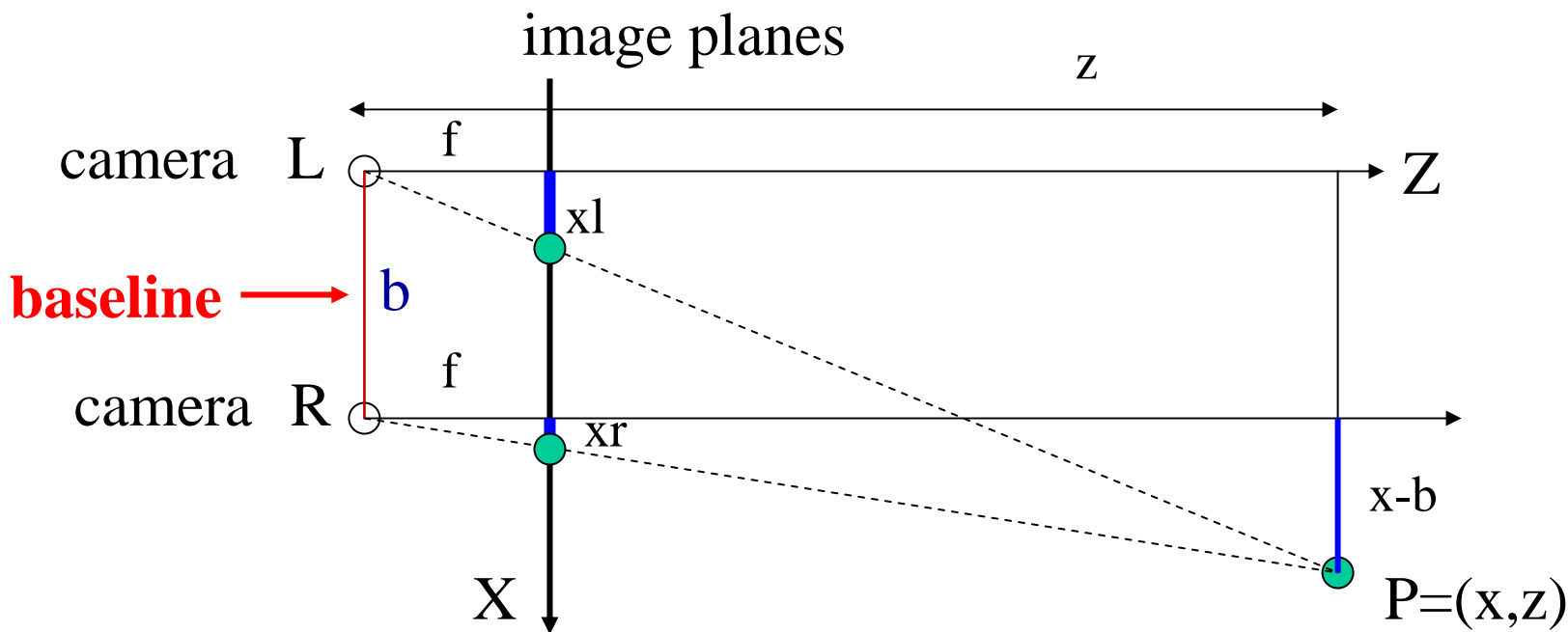
$$\frac{x_i}{f} = \frac{x}{z}$$

(from similar triangles)

(Note: For convenience, we orient  $Z$  axis as above and use  $f$  instead of  $-f$  as in lecture 5)

# Projection for Stereo Images

Simple Model: Optic axes of 2 cameras are parallel



$$\frac{z}{f} = \frac{x}{x_l}$$

$$\frac{z}{f} = \frac{x-b}{x_r}$$

$$\frac{z}{f} = \frac{y}{y_l} = \frac{y}{y_r}$$

$Y$ -axis is perpendicular to the page.

(from similar triangles)

# 3D from Stereo Images: Triangulation

---

For stereo cameras with parallel optical axes, focal length  $f$ , baseline  $b$ , corresponding image points  $(x_l, y_l)$  and  $(x_r, y_r)$ , the location of the 3D point can be derived from previous slide's equations:

$$\text{Depth } z = f*b / (x_l - x_r) = f*b/d$$

$$x = x_l*z/f \quad \text{or} \quad b + x_r*z/f$$

$$y = y_l*z/f \quad \text{or} \quad y_r*z/f$$

This method of determining depth from disparity  $d$  is called **triangulation**.

Note that **depth is inversely proportional to disparity**

---

$$\text{Depth } z = f \cdot b / (x_l - x_r) = f \cdot b / d$$

$$x = x_l \cdot z / f \quad \text{or} \quad b + x_r \cdot z / f$$

$$y = y_l \cdot z / f \quad \text{or} \quad y_r \cdot z / f$$

Two main problems:

1. Need to know focal length  $f$ , baseline  $b$

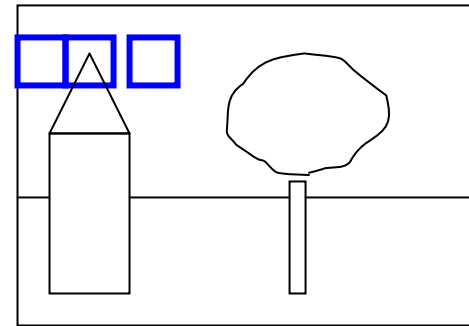
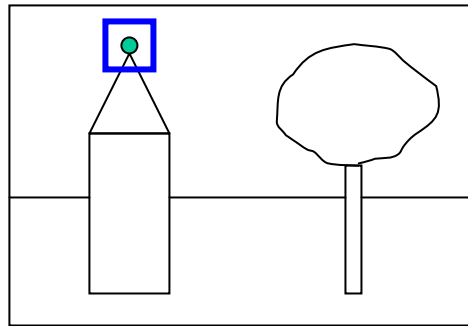
- use prior knowledge or camera calibration

2. Need to find corresponding point  $(x_r, y_r)$  for each  $(x_l, y_l) \Rightarrow$  Correspondence problem

# Solving the stereo correspondence problem

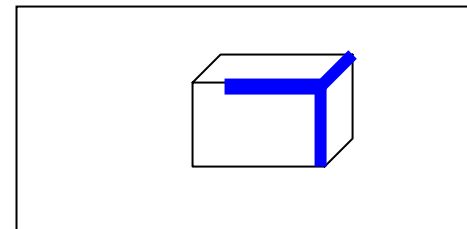
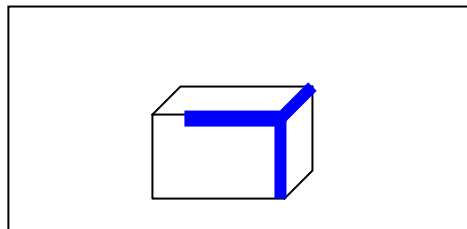
---

1. Cross correlation or SSD using small windows.



dense

2. Symbolic feature matching, usually using segments/corners.



sparse

3. Use the newer interest operators, e.g., SIFT.

sparse

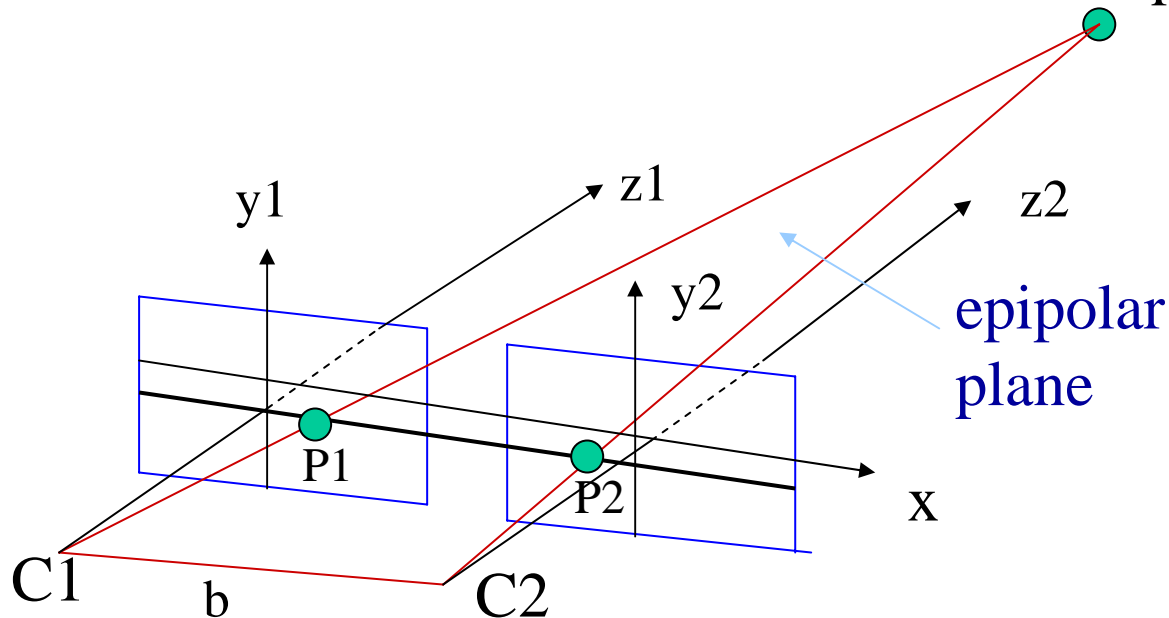
---

Given a point in the left image, do you need to search the entire right image for the corresponding point?



# Epipolar Constraint for Correspondence

Epipolar plane = plane connecting  $C_1$ ,  $C_2$ , and point  $P$



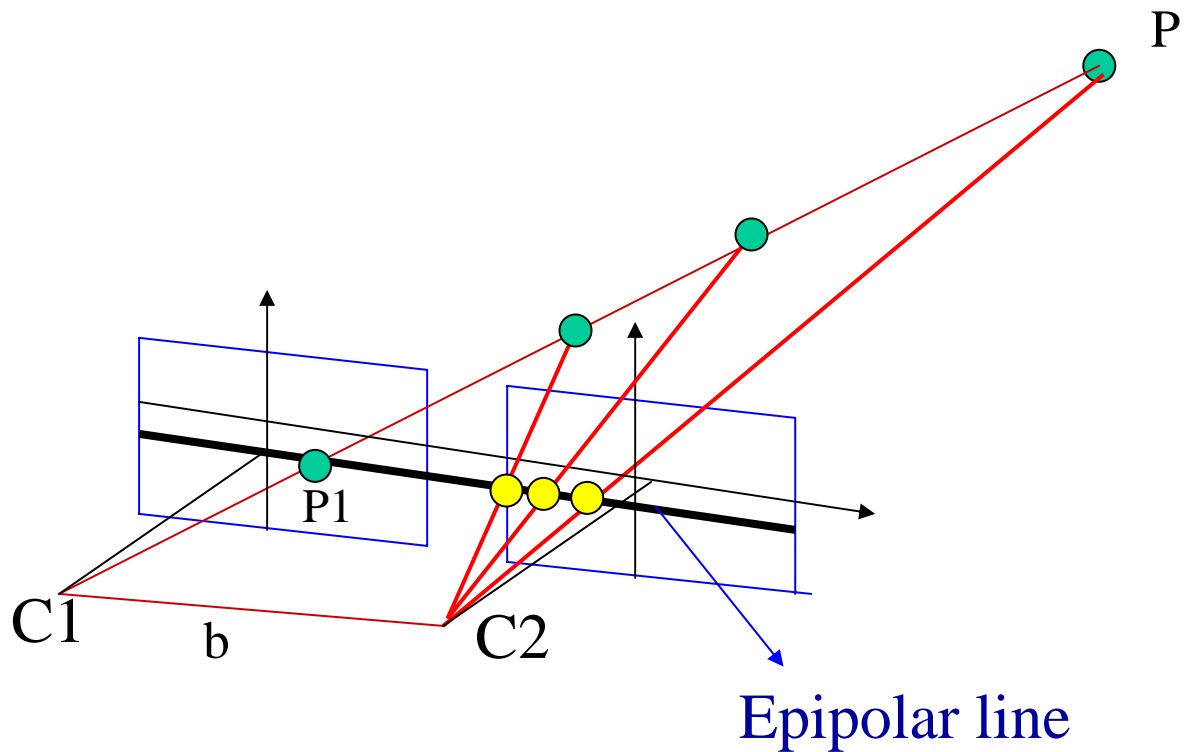
Epipolar plane cuts through image planes forming an epipolar line in each plane

Match for  $P_1$  (or  $P_2$ ) in the other image must lie on epipolar line

# Epipolar Constraint for Correspondence

---

Match for  $P_1$  in the other image must lie on epipolar line  
So need search only along this line

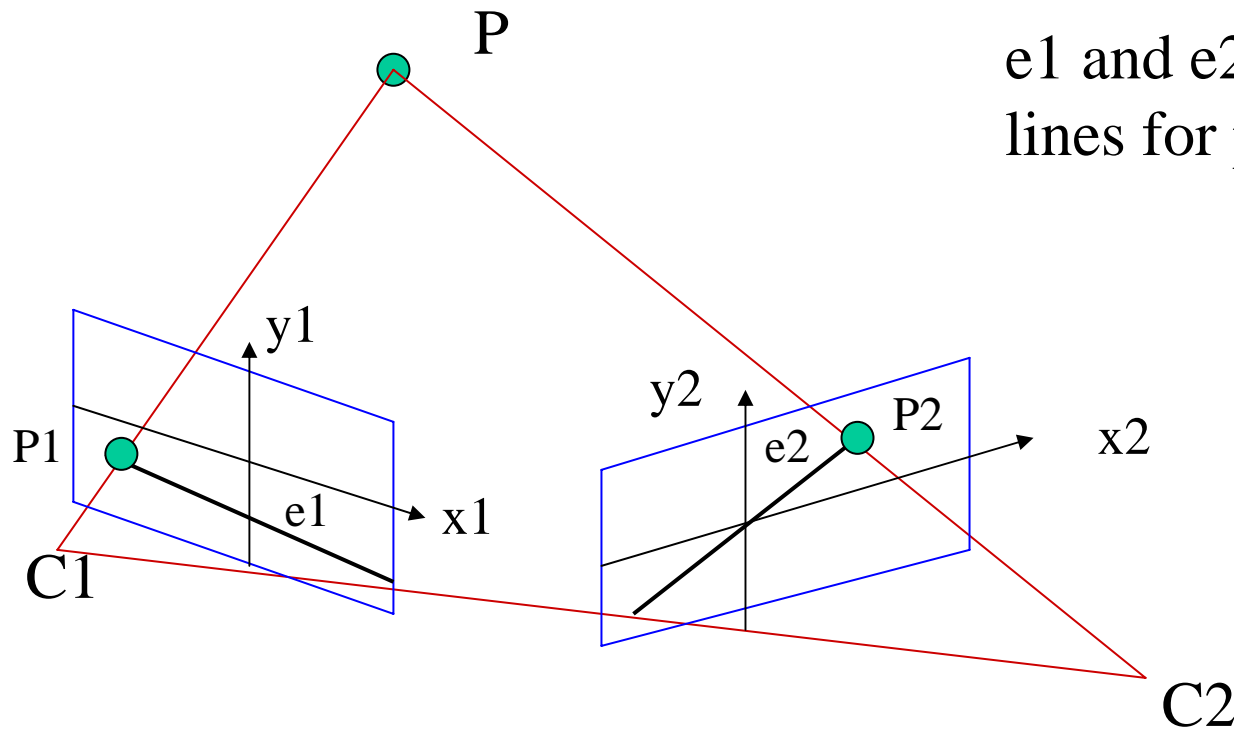


---

What if the optical axes of the 2 cameras are not parallel to each other?

# Epipolar constraint still holds...

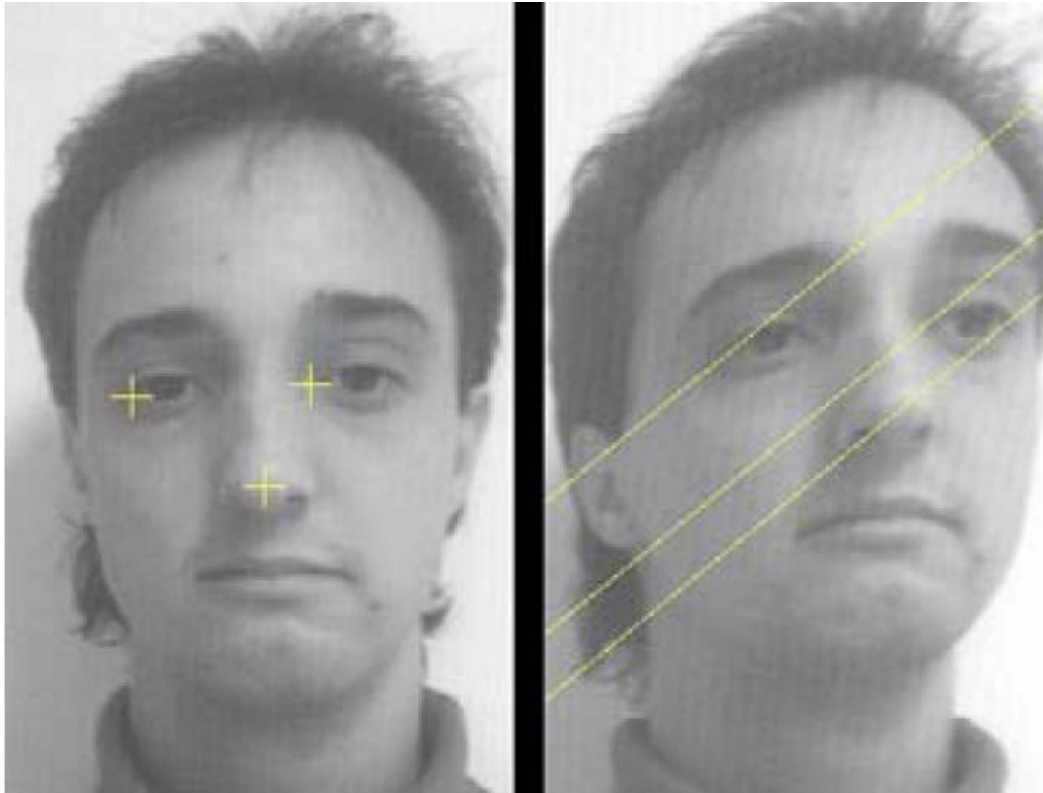
---



But the epipolar lines may no longer be horizontal

# Example

---

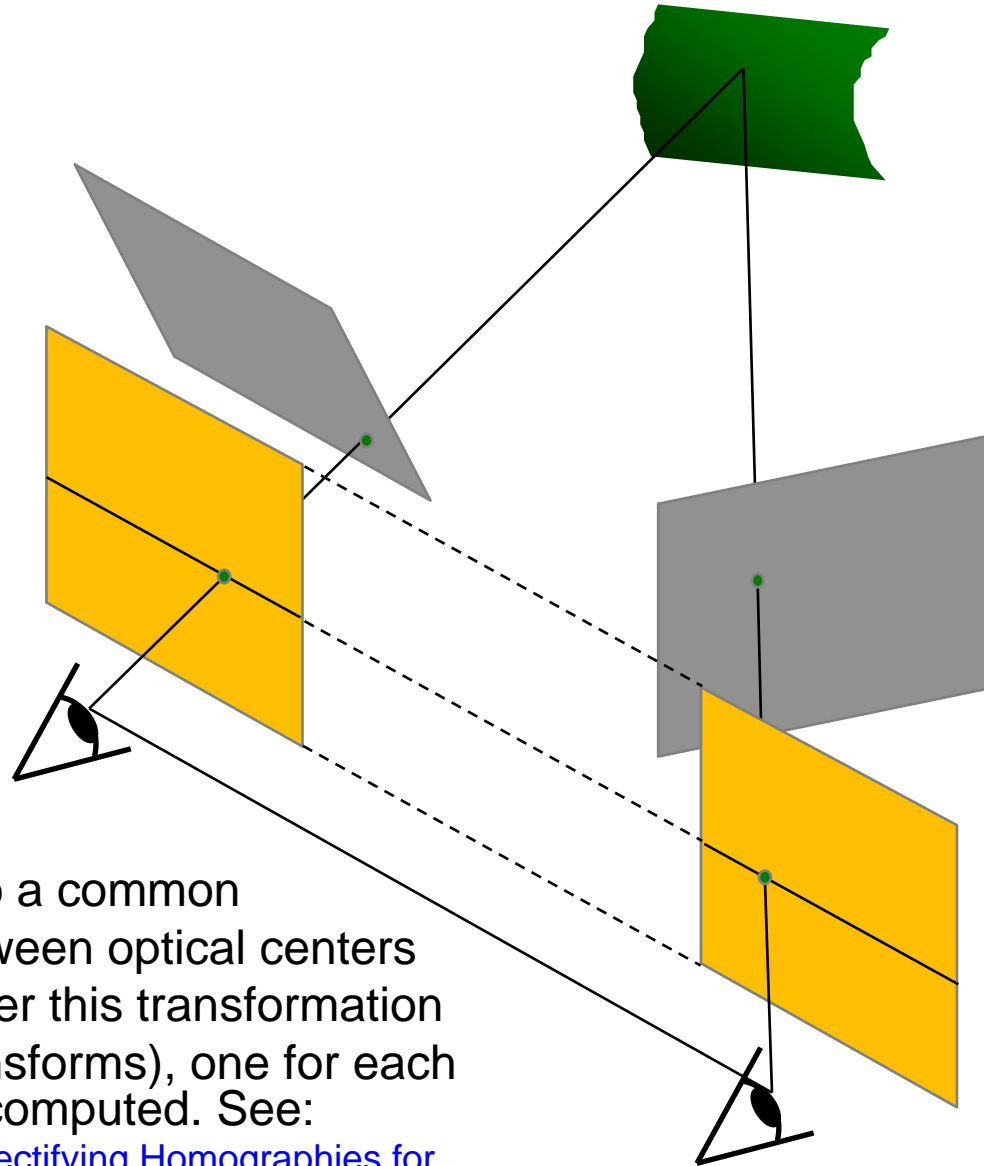


Yellow epipolar lines for the three points shown on the left image

(from a slide by Pascal Fua)

Given a point  $P_1$  in left image on epipolar line  $e_1$ , can find epipolar line  $e_2$  provided we know relative orientations of cameras  $\Rightarrow$  Requires camera calibration (see lecture 5)

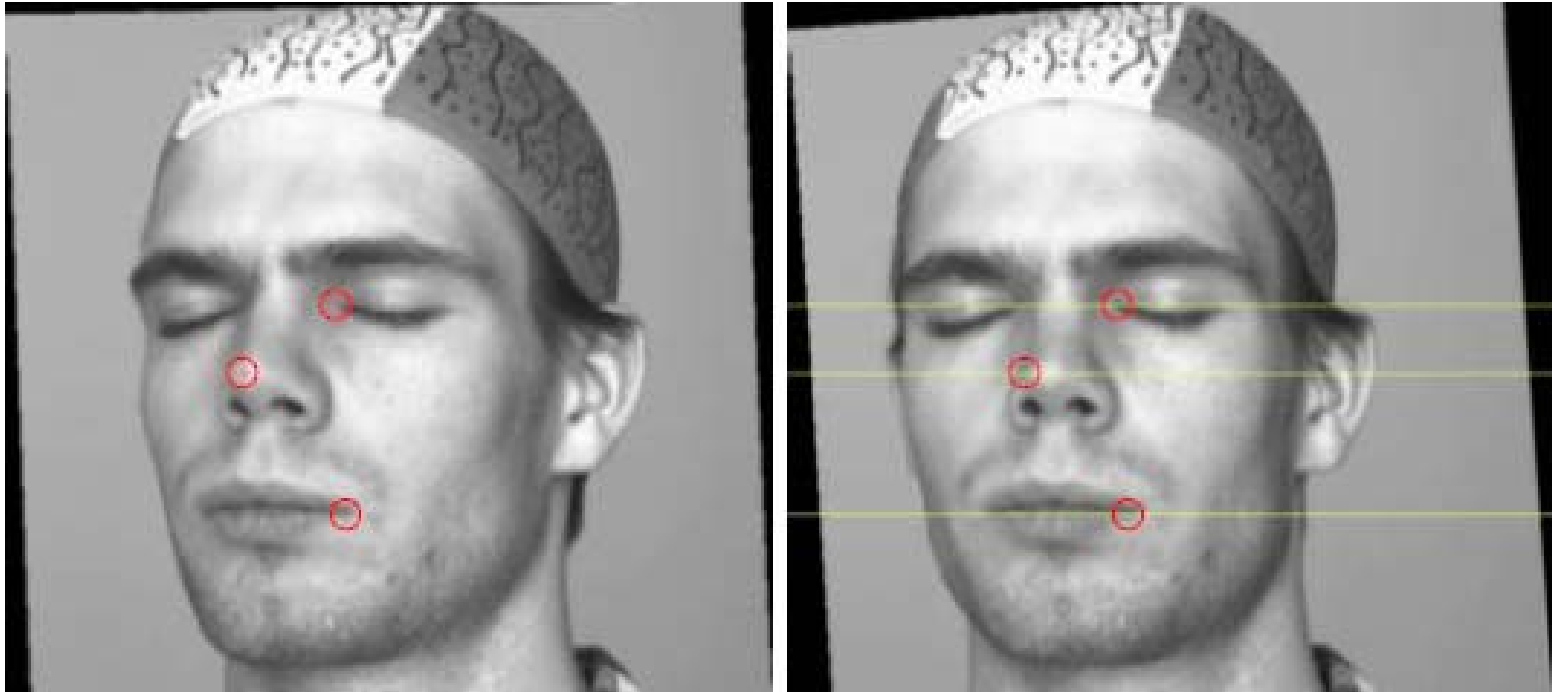
# Alternate approach: Stereo image rectification



- Reproject image planes onto a common plane parallel to the line between optical centers
- Epipolar line is horizontal after this transformation
- Two homographies (3x3 transforms), one for each input image reprojection, is computed. See:
  - C. Loop and Z. Zhang. [Computing Rectifying Homographies for Stereo Vision](#). IEEE Conf. Computer Vision and Pattern Recognition, 1999.

# Example

---

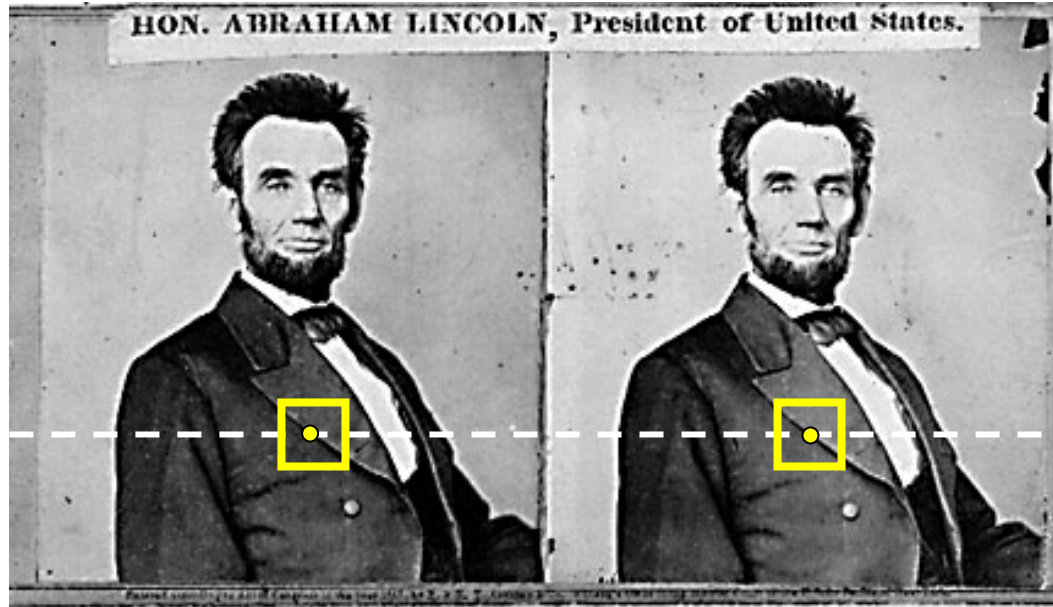


After rectification, need only search for matches along horizontal scan line

(adapted from slide by Pascal Fua)

# Your basic stereo algorithm

---



For each epipolar line

For each pixel in the left image

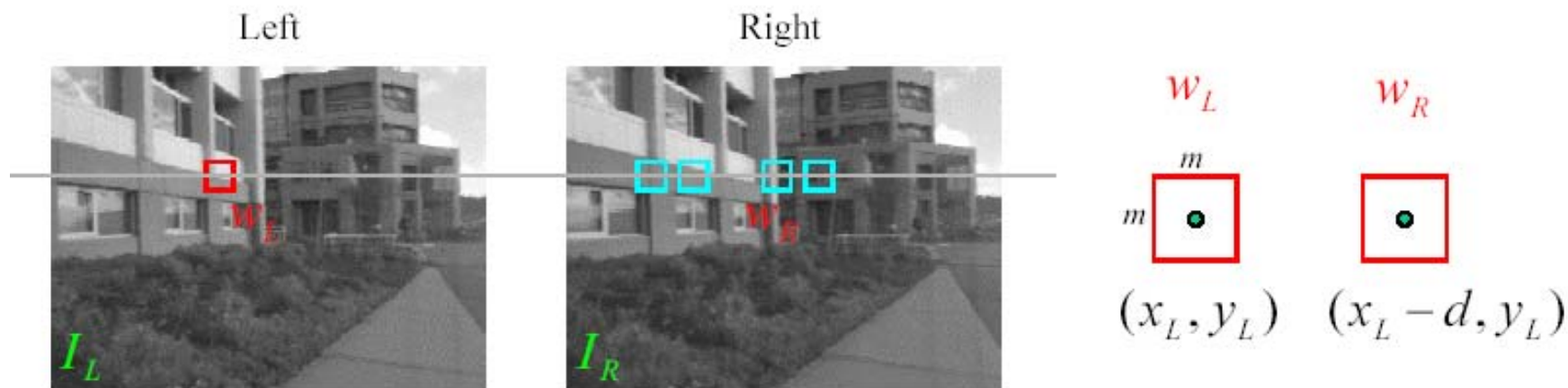
- compare with every pixel on same epipolar line in right image
- pick pixel with minimum match cost

Improvement: match *windows*



# Matching using Sum of Squared Differences (SSD)

---



$w_L$  and  $w_R$  are corresponding  $m$  by  $m$  windows of pixels.

We define the window function :

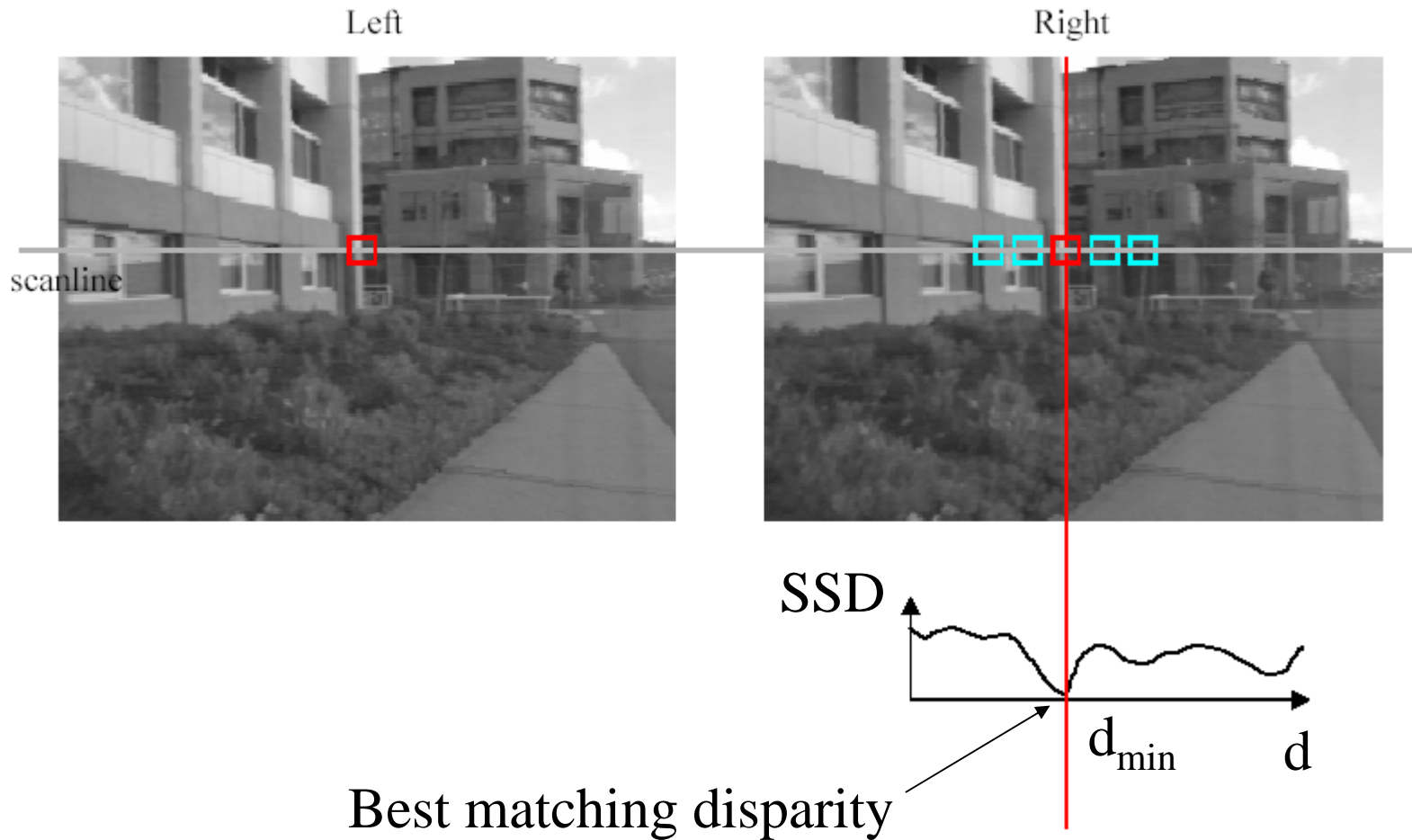
$$W_m(x, y) = \{u, v \mid x - \frac{m}{2} \leq u \leq x + \frac{m}{2}, y - \frac{m}{2} \leq v \leq y + \frac{m}{2}\}$$

The SSD cost measures the intensity difference as a function of disparity :

$$C_r(x, y, d) = \sum_{(u,v) \in W_m(x,y)} [I_L(u, v) - I_R(u - d, v)]^2$$

# Stereo matching based on SSD

---



# Problems with window size

---



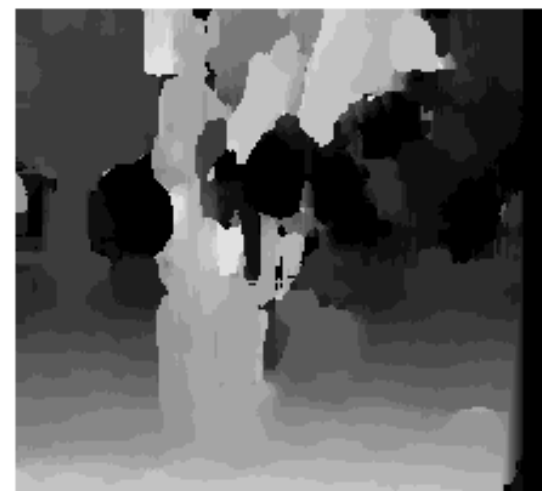
Input stereo pair



$W = 3$

## Effect of window size $W$

- Smaller window
  - + Good precision, more detail
  - Sensitive to noise
- Larger window
  - + Robust to noise
  - Reduced precision, less detail



$W = 20$

# Example depth from stereo results

---

- Data from University of Tsukuba



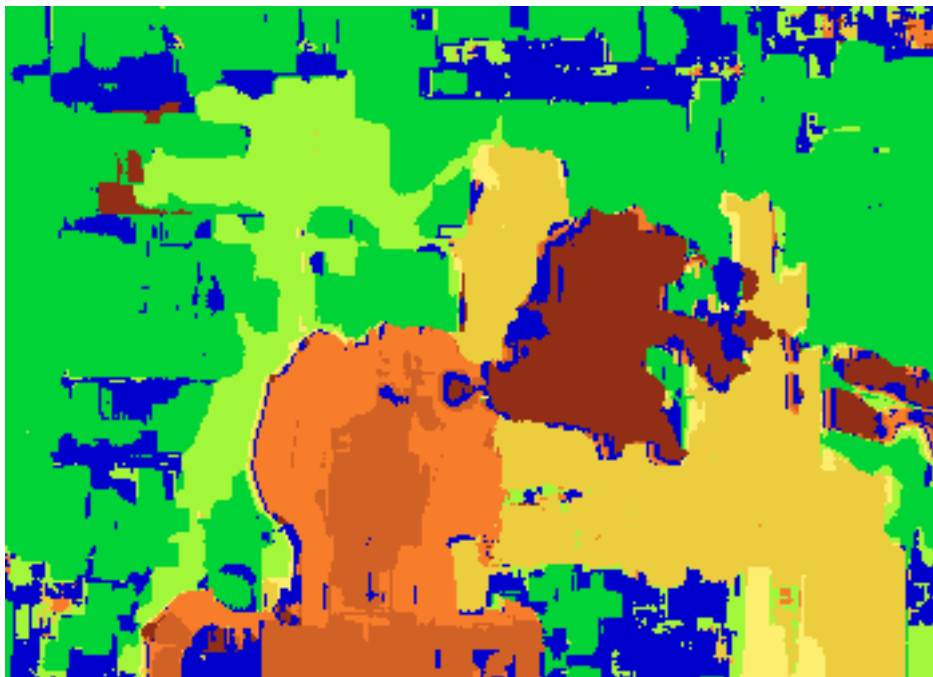
Scene



Ground truth

# Results with window-based stereo matching

---



Window-based matching  
(best window size)



Ground truth

# Better methods exist...

---



State of the art method:

Boykov et al., [Fast Approximate Energy Minimization via Graph Cuts](#),  
International Conference on Computer Vision, 1999

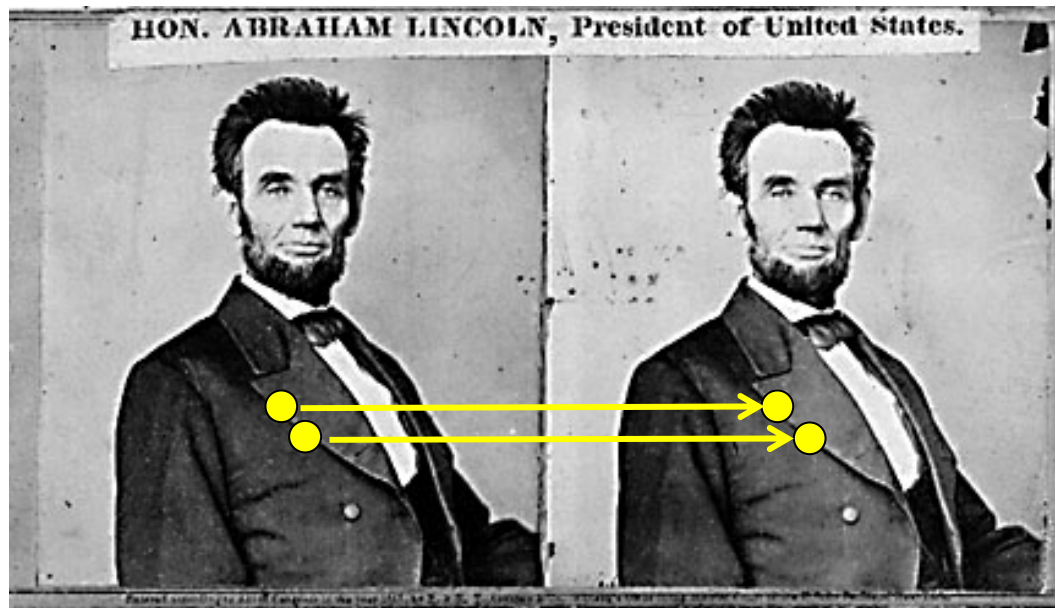


Ground truth

For the latest and greatest: <http://www.middlebury.edu/stereo/>

# State of the art: Stereo as energy minimization

---



What defines a good stereo correspondence?

1. Match quality
  - Want each pixel to find a good match in the other image
2. Smoothness
  - If two pixels are adjacent, they should (usually) be displaced about the same amount i.e., have similar disparities

# Stereo as energy minimization

---

## Expressing this mathematically

### 1. Match quality

- Want each pixel to find a good match in the other image

$$\text{matchCost} = \sum_{x,y} \|I(x, y) - J(x + d_{xy}, y)\|$$

### 2. Smoothness

- If two pixels are adjacent, they should have similar disparities

$$\text{smoothnessCost} = \sum_{\text{neighbor pixels } p,q} |d_p - d_q|$$

We want to minimize  $\text{Energy} = \text{matchCost} + \text{smoothnessCost}$



# Stereo as energy minimization

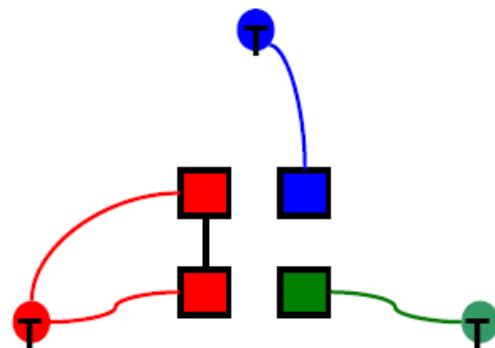
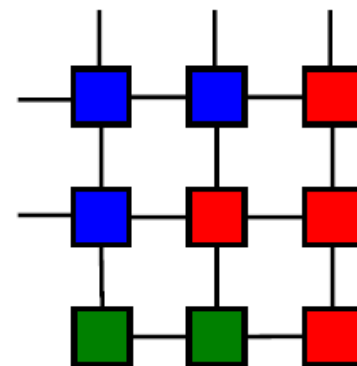
---

We want to minimize:

$$Energy = matchCost + smoothnessCost$$

- This is a special type of energy function known as an MRF (Markov Random Field)
  - Effective and fast algorithms have been recently developed:
    - » Graph cuts, belief propagation....
    - » for more details (and code): <http://vision.middlebury.edu/MRF/>

Image as a graph with disparity labels



Min-cost graph cut yields a labeling of each pixel with best disparity

# Stereo reconstruction pipeline

---

## Steps

- Calibrate cameras
- Rectify images
- Compute disparity
- Estimate depth

## What will cause errors?

- Camera calibration errors
- Poor image resolution
- Occlusions
- Violations of brightness constancy (specular reflections)
- Large motions
- Low-contrast image regions

# Example Application: Robot Navigation

---

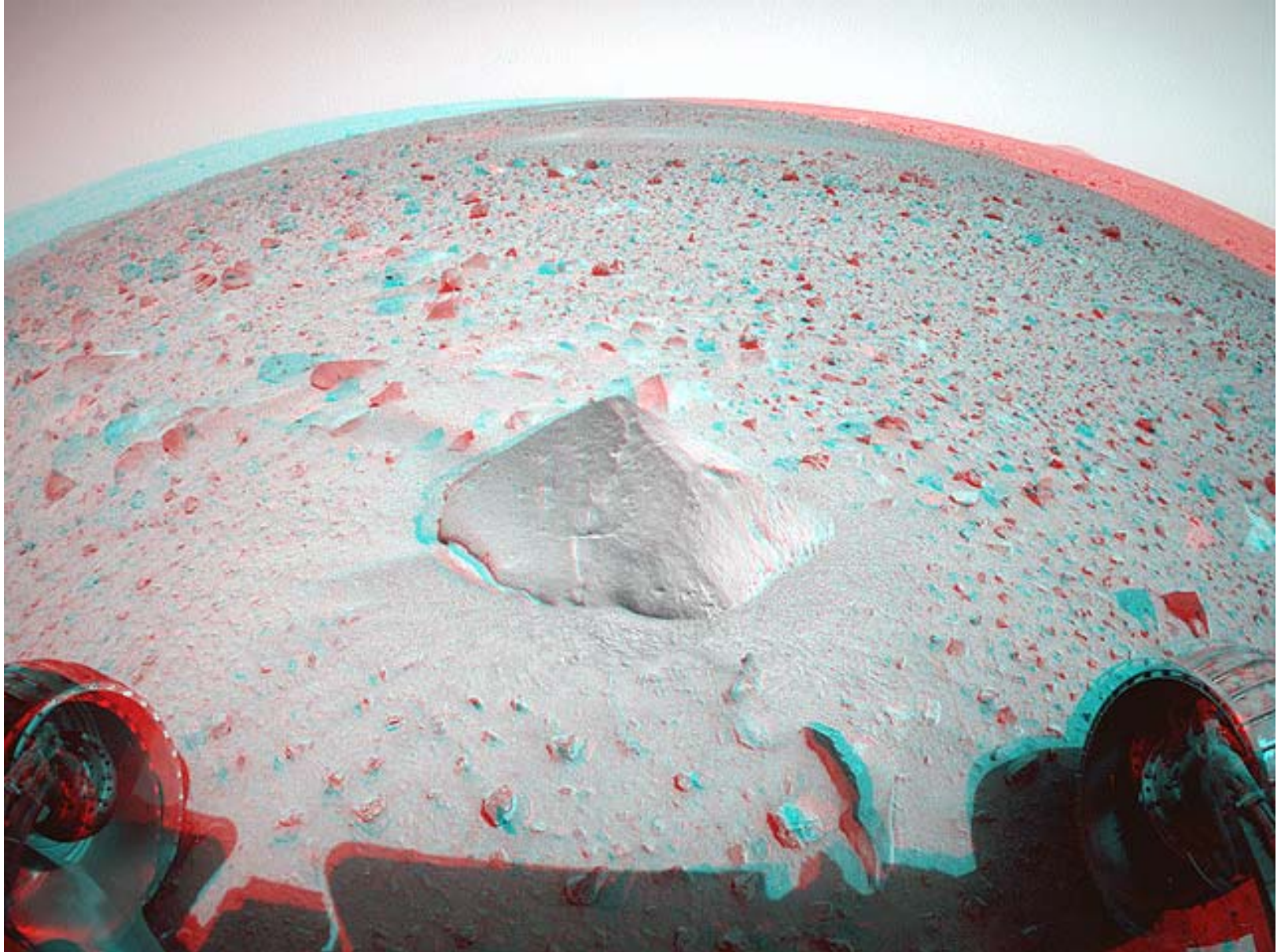


[Nomad robot](http://www.frc.ri.cmu.edu/projects/meteorobot/index.html) searches for meteorites in Antarctica  
<http://www.frc.ri.cmu.edu/projects/meteorobot/index.html>

Stereo also used in Mars Rover (Clark Olson's guest lecture)

# Anaglyph from Mars Rover

---



---

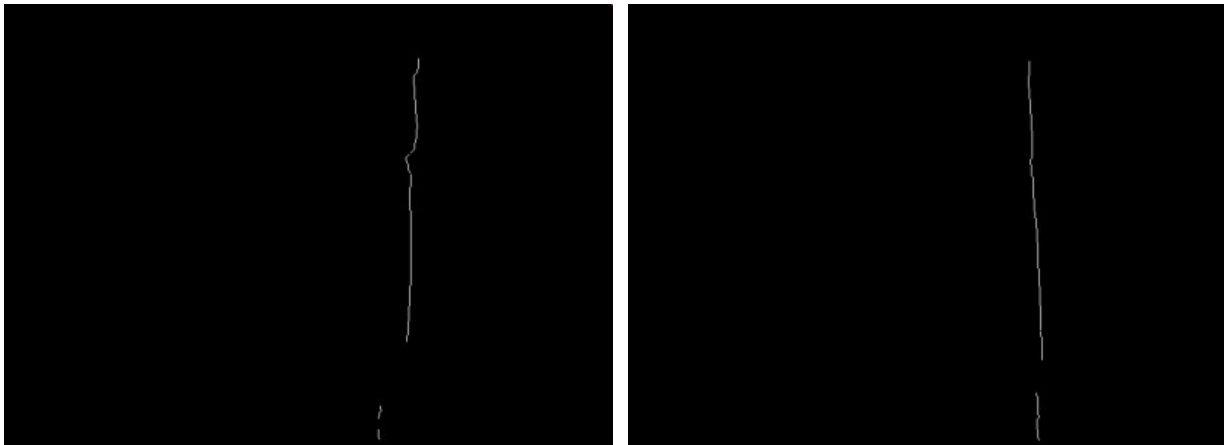
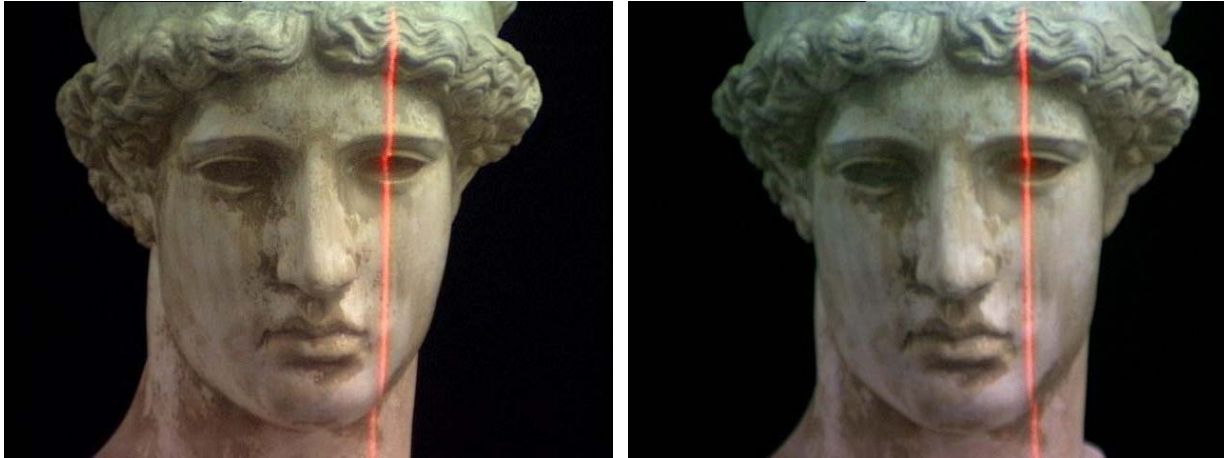
What if 3D object has little or no texture?

Matching points might be difficult or impossible

Can we still recover depth information?

# Idea: Use structured light!

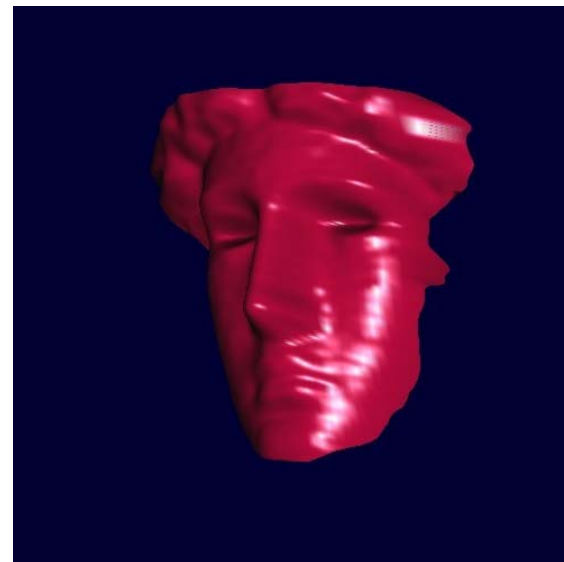
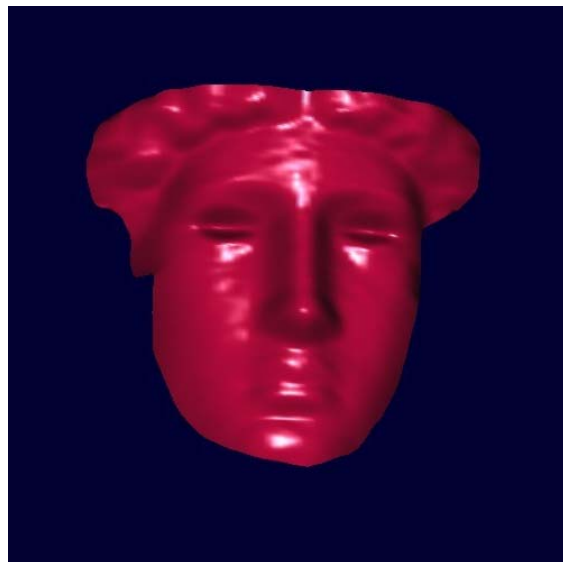
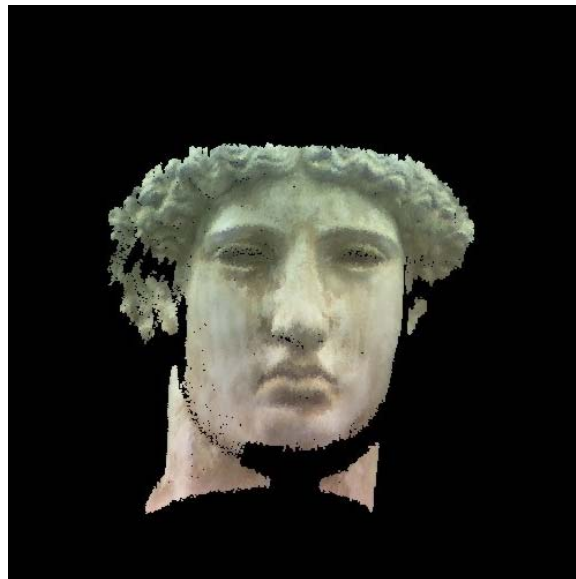
---



Disparity between laser points on the same scanline in the images determines the 3-D coordinates of the laser point on object

# Recovered 3D Model

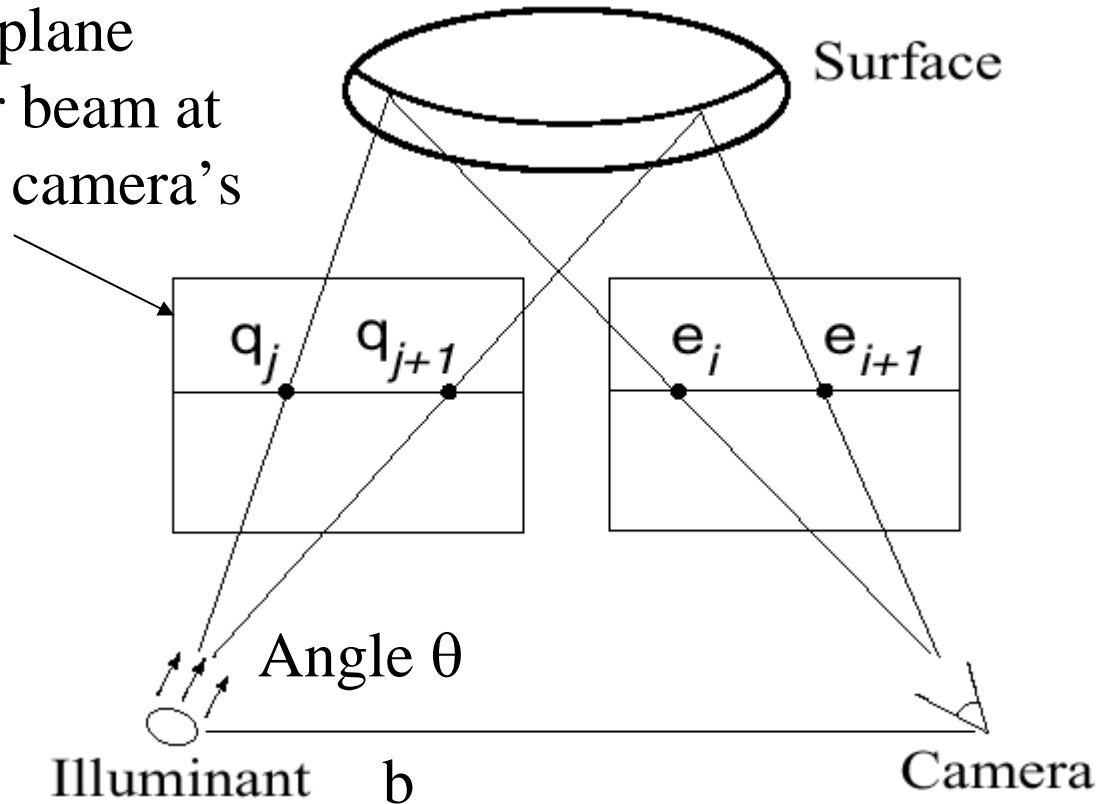
---



# Actually, we can make do with just 1 camera

---

Virtual “image” plane  
intersecting laser beam at  
same distance as camera’s  
image plane

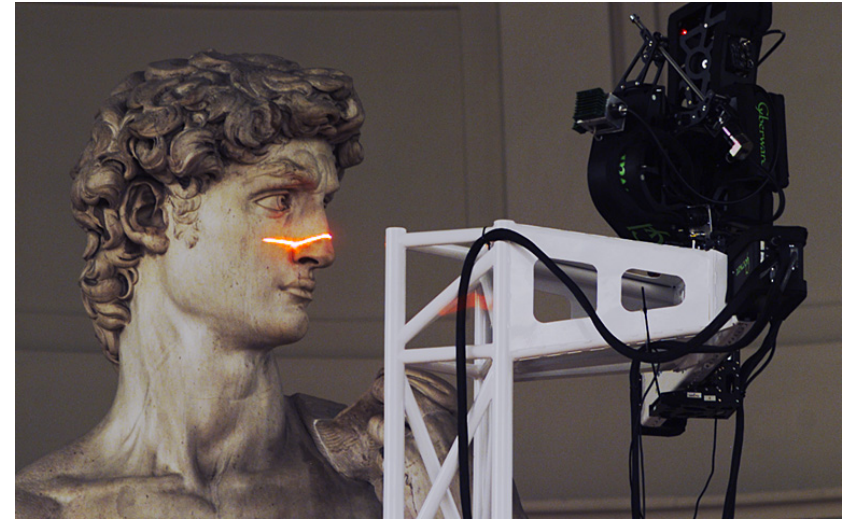
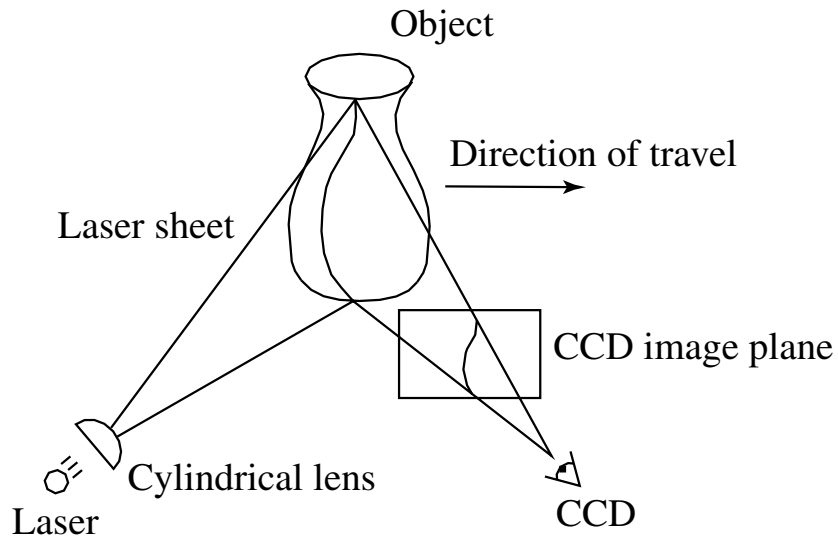


From calibration of both camera and light projector, we can compute 3D coordinates laser points on the surface



# The Digital Michelangelo Project

---



<http://graphics.stanford.edu/projects/mich/>

## Optical triangulation

- Project a single stripe of laser light
- Scan it across the surface of the object

# Laser scanned models

---



*The Digital Michelangelo Project, Levoy et al.*

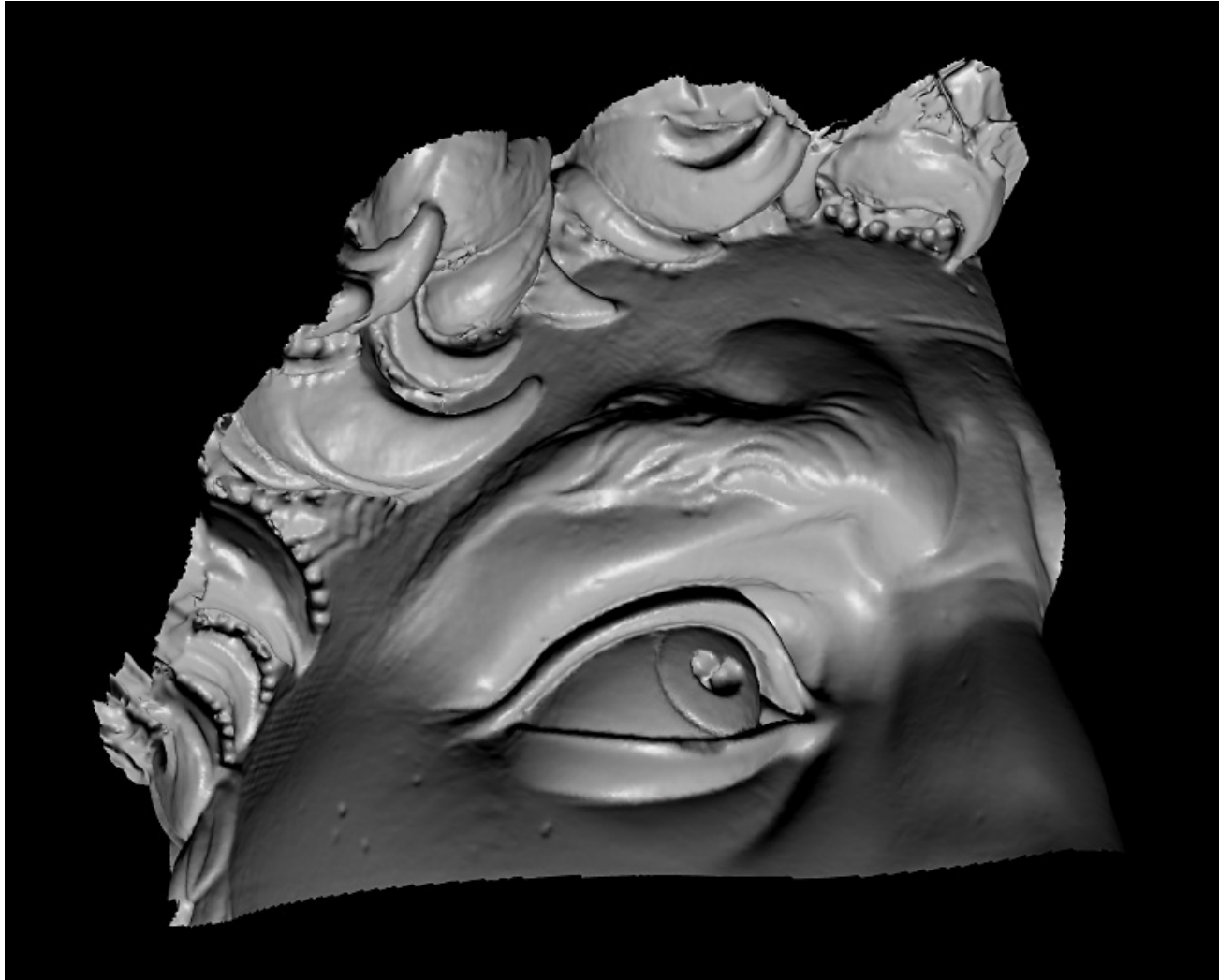
# Laser scanned models



*The Digital Michelangelo Project, Levoy et al.*

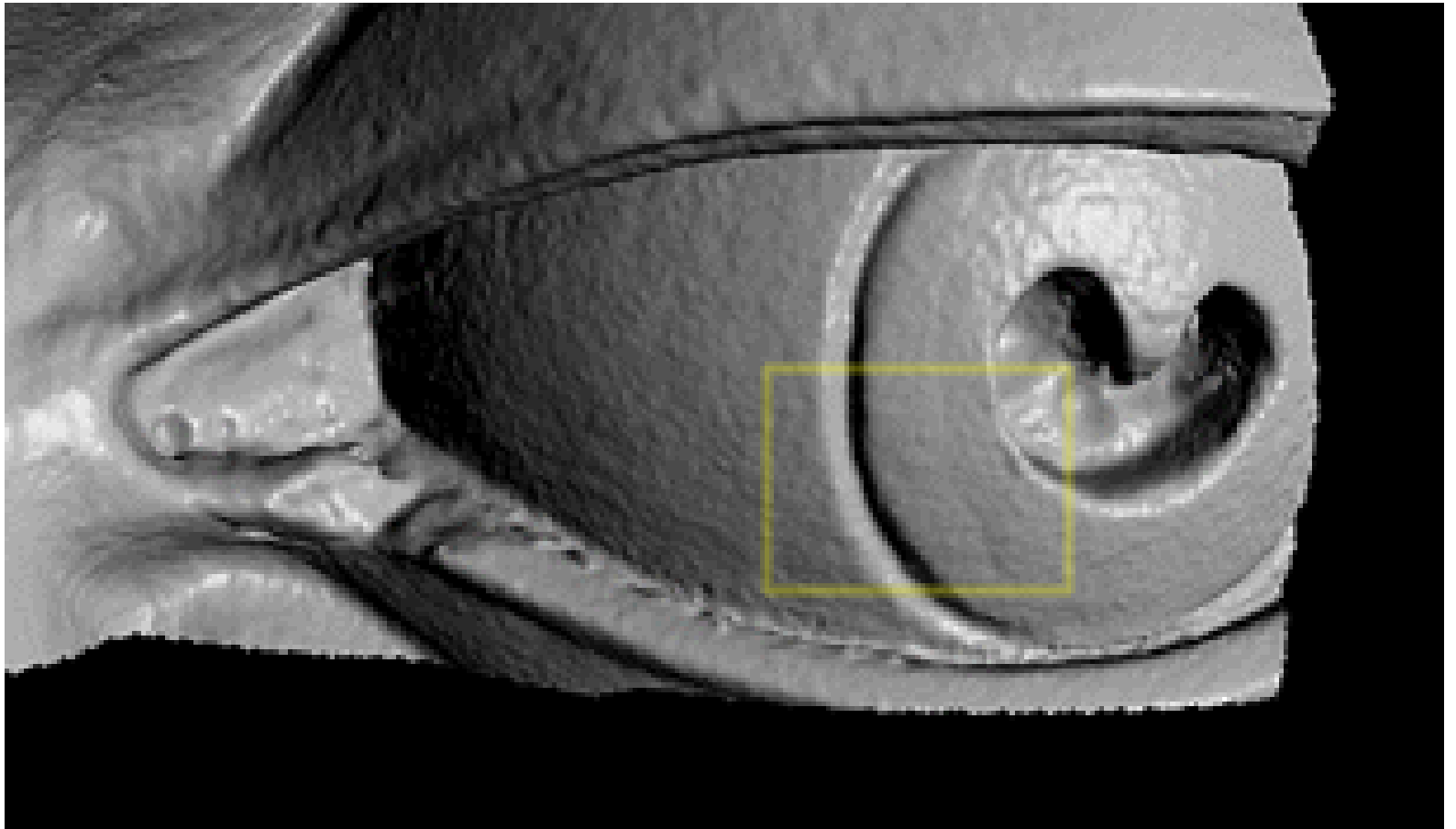
# Laser scanned models

---



*The Digital Michelangelo Project, Levoy et al.*

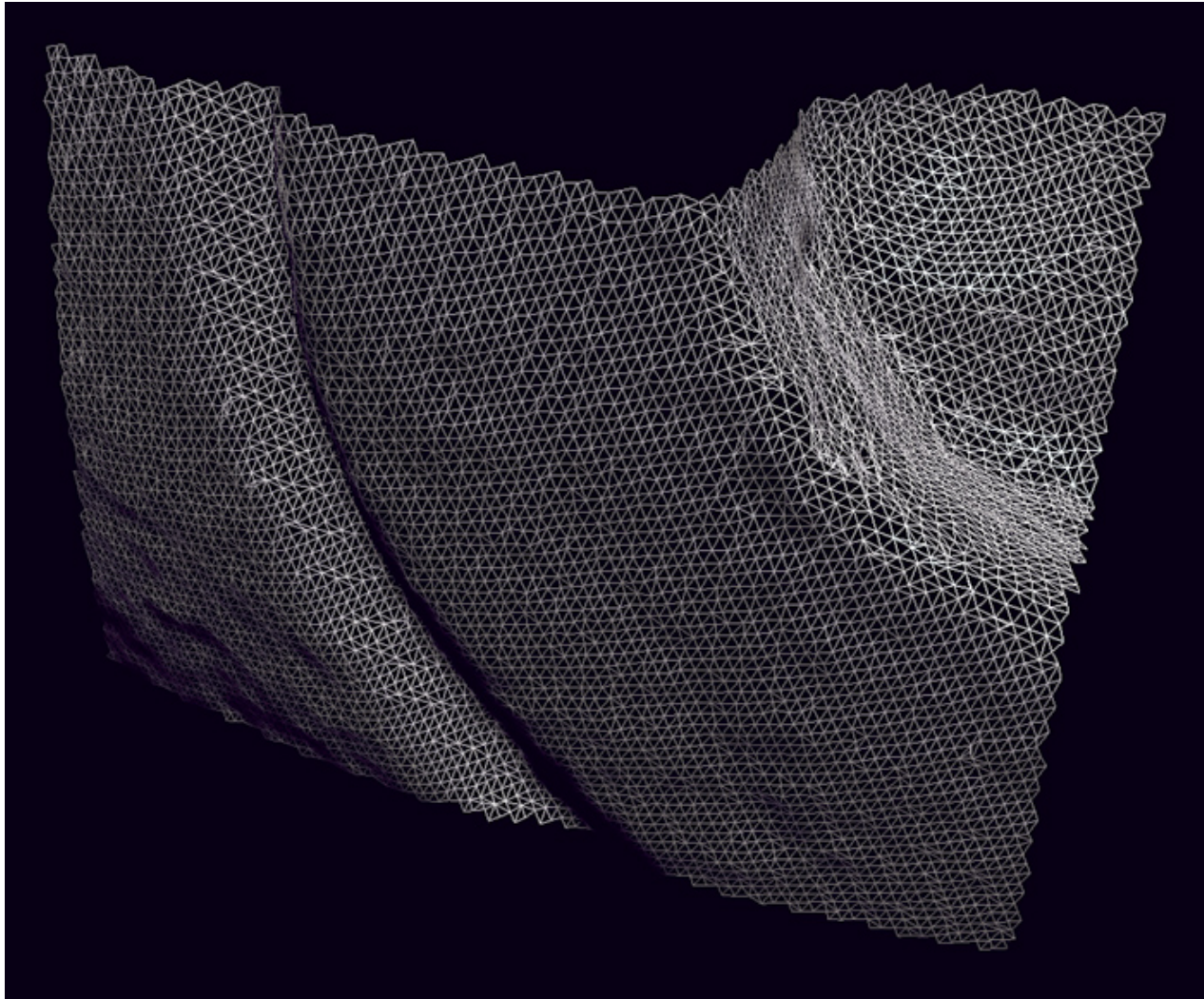
# Laser scanned models



*The Digital Michelangelo Project, Levoy et al.*

# Laser scanned models

---



*The Digital Michelangelo Project, Levoy et al.*

# A cool stereo application: Video View Interpolation

---

<http://research.microsoft.com/users/larryz/videoviewinterpolation.htm>



---

Now, for Project 2 voting results...



# 3<sup>rd</sup> Place Winners (3-way tie)

---

Alex Eckerman and Mike Chung (52 votes)



John Lyon and James George (52 votes)



Aron Ritchie and Andrew Reusch (52 votes)



# 2<sup>nd</sup> Place Winner

---

Paramjit Singh Sandhu and Zhen Wang (63 votes)



---

Drumroll please...

# 1<sup>st</sup> Place Winner

---

Brice Johnson and Will Johnson (68 votes)



# Next Time: Guest lecture by Richard Ladner

---



Things to do:

- Work on Project 4